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On the p-fold Well-posedness of Higher Order Abstract Cauchy Problem

Habiba Toumi*
Laboratory LAMAHIS, University Badji Mokhtar
Annaba 23000, Algeria
Ahmed Nouar†
Laboratory LAMAHIS, University 20 Aût 1955
Skikda 21000, Algeria

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Abstract. In this paper, we establish sufficient conditions for the p-fold well-posedness of higher-order abstract Cauchy problem. These conditions are expressed in terms of decay of some auxiliary pencils derived from the characteristic pencil for the operational differential equation considered. In particular, this paper improves important and interesting work.

Keywords: abstract Cauchy problems, integrated semi-groups, well-posedness.

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1. Introduction

The development of functional analysis and the theory of linear operators appeared at the beginning of the 20th century and had a great influence on the study of ordinary differential equations, partial differential equations, and boundary value problems. In particular, many results concerning the general theory of operator pencils has marked a rapid development (see, for example, [5, 9, 16] and the related sources). This theory is related to the study of boundary values problems for operator-differential equations and higher-order abstract Cauchy problems. Since integrated semi-groups were introduced at the end of the 1980s, it has become possible to deal with ill-posed first-order abstract Cauchy problems (see, [4, 7, 14]). Many authors have made series of direct investigations on the abstract Cauchy problem of the second order and higher order. For more information, we can refer to ([6, 11, 13, 15, 17]) and the references cited therein.

The well-posedness the Cauchy problem is a very large classical problem which has been extensively studied in many contexts. For example in [17], Tijun *et al.* presented a concise criteria for C-well-posedness and analytic well-posedness of the complete second order Cauchy problem.

In [18], Vlasenko and others studied the p-fold well-posedness of the higher order abstract Cauchy problem of the following form:

$$\sum_{k=0}^{n} A_k \frac{d^k u}{dt^k} = 0, \qquad t > 0, \tag{1.1}$$

$$u^k(0) = u_k, \qquad k = 0, \ldots, n-1, \tag{1.2}$$

*toumihabiba80@gmail.com, h.toumi@univ-skikda.dz https://orcid.org/0009-0003-2332-3689
†hamdensk97@gmail.com

where A_k $(k = 0, \dots, n)$ are closed linear operators that act from the complex Banach space $\mathcal{X}$ into the complex Banach space $\mathcal{Y}$. We denote the norms on $\mathcal{X}$ and $\mathcal{Y}$ by $\|\|_{\mathcal{X}}$ and $\|\|_{\mathcal{Y}}$ respectively.

They obtained well-posedness conditions, which characterize the continuous dependence of solutions and their derived on the initial data.

It should be noted that the higher order abstract Cauchy problems of the form (1.1), (1.2) have been studied by many authors, since they describe several models derived from natural phenomena, such as description of vibrations (see, [1, 2, 10]), viscoelastic pipeline in [12]. This means that it deserves to study the p-fold well-posedness and exponentially p-fold well-posedness of a higher order abstract Cauchy problem. The abstract Cauchy problem for the higher order has been extensively investigated by many authors (see, [13, 18, 19] and the references therein). We refer to some researche which relates directly to our work (see, [18, 19]).

In [18], Vlasenko *et al.* investigated the equation (1.1) with a characteristic polynomial and its resolvent

$$\mathcal{P}(\lambda) = \sum_{k=0}^{n} A_k \lambda^k, \quad R(\lambda) = \mathcal{P}^{-1}(\lambda), \tag{1.3}$$

the authors derived some new conditions represented in the estimates using $R(\lambda)$ to ensure the correct setting of the p-fold of the problem (1.1), (1.2).

Motivated by this work, this paper aims to establish new sufficient conditions which guarantee the p-fold well-posedness and exponentially p-fold well-posedness of the problem (1.1), (1.2). These conditions are expressed in terms of the decay of the auxiliary pencils $\mathcal{P}_j(\lambda)$ and $\mathcal{Q}_j(\lambda)$, which are respectively defined by:

$$\mathcal{P}_j(\lambda) = \sum_{k=0}^{j} A_k \lambda^k, \tag{1.4}$$

$$\mathcal{Q}_j(\lambda) = \sum_{k=j+1}^{n} A_k \lambda^{k-j-1}, \tag{1.5}$$

for $j \in \{0, \dots, n-1\}$.

As we mentioned before, the results of this work extend and improve the previously known results. More precisely, we will consider a higher order abstract Cauchy problem and give some new sufficient conditions to ensure p-fold well-posedness and exponentially p-fold well-posedness of the problem (1.1), (1.2). In our analysis, it is not necessary to use all the operators A_k, $k = 1, \dots, n$, which are required in some relevant preliminary work, see [18]. This new feature makes the p-fold well-posedness of the higher order abstract Cauchy more important and useful as well. The results of this article are new and they extend and improve previously known results.

The content of this paper is organized as follows:
Some necessary concepts and preliminaries are reviewed in Section 2. Section 3 is a section of intermediate results, where we show some lemmas which are necessary for our analysis. Finally, in Section 4, we give and prove our main results.

2. Preliminaries

In this section, we briefly recall some notations and definitions which are used throughout the paper.

Let $\mathcal{G}(a, \theta)$ be the sector of the plane defined by

$$\mathcal{G}(a, \theta) = \left\{ \lambda = a + re^{i\varphi}, \ |\varphi| \leqslant \theta, \ a, r > 0, \ \frac{\pi}{2} < \theta < \pi \right\}.$$

In order to discuss the statement of our problem, we need the following definitions.

Definition 2.1 (see [18]). *By solution of the problem* (1.1), (1.2)*, we mean every function* u *which satisfies the conditions:*

(i) $u \in C^n(\mathbb{R}_+^*, \mathcal{X}) \bigcap C^{n-1}(\mathbb{R}_+, \mathcal{X})$*;*

(ii) $A_k u \in C^k(\mathbb{R}_+^*, \mathcal{Y}) \bigcap C^{k-1}(\mathbb{R}_+, \mathcal{Y})$;

(iii) u *satisfies condition* (1.2).

Definition 2.2 (see [18]). *The abstract Cauchy problem* (1.1), (1.2) *is said to be determined if for fixed* $\{u_k\}_0^{n-1}$*, there exists only one solution.*

Definition 2.3 (see [18]). *Let* $p \in \{1, \dots, n\}$*, the problem* (1.1), (1.2) *is said to be p-fold well-posed if every solution* u *verifies:*

$$\left\|u^{p-1}(t)\right\|_{\mathcal{X}} \leqslant q(t) \sum_{k=0}^{n-1} \|u_k\|_{\mathcal{X}}, \ \ t \geqslant 0, \tag{2.1}$$

where q *is a non-negative function from* $\mathbb{R}_+$ *to* $\mathbb{R}_+$.

Definition 2.4 (see [18]). *The problem* (1.1), (1.2) *is said to be exponentially well-posed if* $q(t) = Ce^{\omega t}$*, for* $C \geqslant 0$*, and* $\omega \geqslant 0$.

Lemma 2.5 (Jordan lemma (see [3])). *Let* m *be positive constant and* $Q(z)$ *be a continuous function in the upper half of complex plane, such that for* $|z| \geqslant R$

$$M_R = \max_{z \in \Gamma_R} |Q(z)| \to 0, \ \ R \to \infty,$$

where Γ_R *is the semicircle of* $|z| = R$ *in the upper half of the complex plane. Then*

$$\lim_{R\to\infty} \int_{\Gamma_R} e^{imz} \mathcal{Q}(z)\, dz = 0.$$

Theorem 2.6 (see [16]). *Let Banach space* $\mathcal{X}$ *and* A *is closed linear operators from* $\mathcal{X}$ *into* $\mathcal{X}$*, such that* $\|A\| \leqslant q < 1$*. Then* $(I + A)$ *is reversible and* $(I + A)^{-1}$ *is bounded operator.*

3. Intermediate results

In this section, we establish three fundamental lemmas and for convenience we present their proofs. The following lemmas are useful for the proof of our main results in the last section.

Lemma 3.1. *If there exist* $j \in \{0, \dots, n-1\}$ *such that*
$H_1)$ $\mathcal{G}(a, \theta) \subset \rho(\mathcal{Q}_j(\lambda))$,
$H_2)$ $\left\|\mathcal{Q}_j^{-1}(\lambda) A_k x\right\|_{\mathcal{X}} \leqslant C |\lambda|^{j+1} e^{\sigma|\lambda|} \|x\|_{\mathcal{X}}, \ \ k = 1, \dots, n,$
$H_3)$ $\left\|\mathcal{Q}_j^{-1}(\lambda) A_k x\right\|_{\mathcal{X}} \leqslant C_j |\lambda|^{q_{k_j}} \|x\|_{\mathcal{X}}, \ k = 0, \dots, j,$ *with* $\dfrac{\ln C_j}{\ln(a \sin\theta)} < q_{k_j} < j - k + 1,$
then,

$$\mathcal{G}(a, \theta) \subset \rho(\mathcal{P}(\lambda)),$$

and

$$\left\|\mathcal{P}^{-1}(\lambda) A_k x\right\|_{\mathcal{X}} \leqslant Ce^{\sigma|\lambda|} \|x\|_{\mathcal{X}}, \ k = 1, \dots, n. \tag{3.1}$$

Proof. If $\lambda \in \mathcal{G}(a,\theta)$, then $\lambda \in \rho(\mathcal{Q}_j(\lambda))$, and $\mathcal{Q}_j(\lambda)$ are invertible, $j = 0,\dots,n-1$. In fact, by assumptions (1.3), (1.4), and (1.5), it can be easily write:

$$\begin{aligned}\mathcal{P}(\lambda) &= \lambda^{j+1}\mathcal{Q}_j(\lambda) + \mathcal{P}_j(\lambda) = \\ &= \lambda^{j+1}\mathcal{Q}_j(\lambda)\left[I + \lambda^{-j-1}\mathcal{Q}_j^{-1}(\lambda)\mathcal{P}_j(\lambda)\right].\end{aligned}$$

By using the Theorem 2.6, we show that the operator (1.3) is invertible.
By (H_3), we have

$$\begin{aligned}\left\|\lambda^{-j-1}\mathcal{Q}_j^{-1}(\lambda)\mathcal{P}_j(\lambda)\right\|_{\mathcal{X}} &\leqslant \sum_{k=0}^{j}|\lambda|^{k-j-1}\left\|\mathcal{Q}_j^{-1}(\lambda)A_k\right\|_{\mathcal{X}} \leqslant \\ &\leqslant C_j\sum_{k=0}^{j}|\lambda|^{k-j-1+q_{k_j}}.\end{aligned}$$

If we let $N_j = \max(j+1-k-q_{k,j})$, then

$$\left\|\lambda^{-j-1}\mathcal{Q}_j^{-1}(\lambda)\mathcal{P}_j(\lambda)\right\|_{\mathcal{X}} \leqslant \frac{C_j(j+1)}{|\lambda|^{N_j}}.$$

From assumption (H_3), we decuce

$$N_j > \frac{\ln C_j}{\ln(a\sin\theta)}.$$

From the above discussion, we can get

$$\left\|\lambda^{-j-1}\mathcal{Q}_j^{-1}(\lambda)\mathcal{P}_j(\lambda)\right\|_{\mathcal{X}} < 1. \tag{3.2}$$

We deduce that the operator pencil

$$I + \lambda^{-j-1}\mathcal{Q}_j^{-1}(\lambda)\mathcal{P}_j(\lambda),$$

is invertible as well as $\mathcal{P}(\lambda)$. Consequently

$$\mathcal{P}^{-1}(\lambda) = \lambda^{-j-1}\left(I + \lambda^{-j-1}\mathcal{Q}_j^{-1}(\lambda)\mathcal{P}_j(\lambda)\right)^{-1}\mathcal{Q}_j^{-1}(\lambda). \tag{3.3}$$

If we put

$$\mathcal{R}_j(\lambda) = \left(I + \lambda^{-j-1}\mathcal{Q}_j^{-1}(\lambda)\mathcal{P}_j(\lambda)\right)^{-1}. \tag{3.4}$$

Make substitution of (3.4) into (3.3), we have

$$\mathcal{P}^{-1}(\lambda) = \mathcal{R}_j(\lambda)\lambda^{-j-1}\mathcal{Q}_j^{-1}(\lambda). \tag{3.5}$$

Then, we conclude

$$\left\|\mathcal{P}^{-1}(\lambda)A_kx\right\|_{\mathcal{X}} \leqslant \|\mathcal{R}_j(\lambda)\|_{\mathcal{X}}|\lambda|^{-j-1}\left\|\mathcal{Q}_j^{-1}(\lambda)A_kx\right\|_{\mathcal{X}}.$$

Using assumption (H_2), we get

$$\left\|\mathcal{P}^{-1}(\lambda)A_kx\right\|_{\mathcal{X}} \leqslant C\|\mathcal{R}_j(\lambda)\|_{\mathcal{X}}|\lambda|^{-j-1}|\lambda|^{j+1}e^{\sigma|\lambda|}\|x\|_{\mathcal{X}}.$$

From (3.2) and Theorem 2.6, we conclude that $\mathcal{R}_j(\lambda)$ is bounded, i.e, $\exists G_0 > 0$, such that

$$\|\mathcal{R}_j(\lambda)\|_{\mathcal{X}} \leqslant G_0. \tag{3.6}$$

According to (3.6), we can finaly write

$$\left\|\mathcal{P}^{-1}(\lambda)A_kx\right\|_{\mathcal{X}} \leqslant G_1e^{\sigma|\lambda|}\|x\|_{\mathcal{X}}, \quad k = 1,\dots,n, \tag{3.7}$$

where $G_1 = CG_0$. □

Corollary 3.2. *If the hypotheses* (H_1), (H_2) *and* (H_3) *are fulfilled, then*

$$\left\|\mathcal{P}^{-1}(\lambda)\,\mathcal{Q}_k(\lambda)\right\|_{\mathcal{X}} \leqslant Ce^{\sigma|\lambda|} f_k(|\lambda|)\,, k=1,\ldots,n, \tag{3.8}$$

where

$$f_k(\lambda) = \sum_{i=k+1}^{n} \lambda^{i-k-1}. \tag{3.9}$$

Proof. From (1.5), we can easly get

$$\mathcal{P}^{-1}(\lambda)\,\mathcal{Q}_k(\lambda) = \mathcal{P}^{-1}(\lambda)\sum_{i=k+1}^{n} A_k \lambda^{i-k-1},$$

according to Lemma 3.1, we find

$$\begin{aligned}\left\|\mathcal{P}^{-1}(\lambda)\,\mathcal{Q}_k(\lambda)\right\|_{\mathcal{X}} &\leqslant \sum_{i=k+1}^{n} |\lambda|^{i-k-1}\left\|\mathcal{P}^{-1}(\lambda)\,A_k\right\|_{\mathcal{X}} \leqslant \\ &\leqslant Ce^{\sigma|\lambda|}\sum_{i=k+1}^{n}|\lambda|^{i-k-1} = Ce^{\sigma|\lambda|} f_k(|\lambda|)\,,\end{aligned}$$

where f_k is defined in (3.9).

Lemma 3.3. *If the hypotheses of Lemma 3.1 are satisfied and* u *is a solution of problem* (1.1), (1.2), *then there exists* $M>0$, *such that*

$$\|u^m(t)\|_{\mathcal{X}} \leqslant Me^{(a-a\cos\theta)t}\sum_{k=0}^{n-1}\|u_k\|_{\mathcal{X}}\,, \tag{3.10}$$

for

$$t > -\frac{\sigma}{\cos\theta}, \quad m=0,\ldots,n-1.$$

Proof. If the hypoteses of Lemma 3.1 are satisfied, then relation (3.1) is true and by applying Lemma 1 in [19], we obtain the following representation for solution u and its derivatives:

$$u^m(t) = -\frac{1}{2\pi i}\int_\Gamma \lambda^m e^{\lambda t}\mathcal{P}^{-1}(\lambda)\sum_{k=0}^{n-1}\mathcal{Q}_k(\lambda)\,u_k d\lambda, \tag{3.11}$$

for $t > -\dfrac{\sigma}{\cos\theta}$, and Γ is the boundary of $\mathcal{G}(a,\theta)$.
It follows that

$$\|u^m(t)\|_{\mathcal{X}} \leqslant \frac{C}{2\pi}\int_\Gamma |\lambda|^m e^{tRe\lambda}\left\|\sum_{k=0}^{n-1}\mathcal{P}^{-1}(\lambda)\,\mathcal{Q}_k(\lambda)\,u_k\right\|_{\mathcal{X}} d\lambda. \tag{3.12}$$

As a consequence of Corollary 2.2, we can able to write

$$\|u^m(t)\|_{\mathcal{X}} \leqslant \frac{C}{2\pi}\int_\Gamma |\lambda|^m e^{tRe\lambda}e^{\sigma|\lambda|}\sum_{k=0}^{n-1} f_k(|\lambda|)\,\|u_k\|_{\mathcal{X}}\,d\lambda, \tag{3.13}$$

for $\lambda \in \mathcal{G}(a,\theta)$, where

$$|\lambda| \leqslant a+r, \text{ and } Re\lambda = a + r\cos\theta.$$

By using parametrisation $\lambda = a + re^{i\theta}$, then the estimate (3.13) can be written as follows

$$\|u^m(t)\|_{\mathcal{X}} \leqslant \frac{C}{\pi}\int_0^\infty (a+r)^m e^{t(a+r\cos\theta)} e^{\sigma(a+r)} \sum_{k=0}^{n-1} f_k(r) \|u_k\|_{\mathcal{X}}\, dr,$$

therefore

$$\begin{aligned} \|u^m(t)\|_{\mathcal{X}} &\leqslant \frac{C}{\pi} e^{t(a-a\cos\theta)} \int_0^\infty e^{(\sigma+t\cos\theta)(a+r)} \sum_{k=0}^{n-1} (a+r)^m f_k(r+a) \|u_k\|_{\mathcal{X}}\, dr \leqslant \\ &\leqslant \frac{C}{\pi} e^{t(a-a\cos\theta)} \sum_{k=0}^{n-1} \int_0^\infty e^{(\sigma+t\cos\theta)(a+r)} (a+r)^m f_k(r+a) \|u_k\|_{\mathcal{X}}\, dr. \end{aligned} \tag{3.14}$$

If we let

$$\mathcal{F}_k(t) = \int_0^\infty e^{(\sigma+t\cos\theta)(a+r)} (a+r)^m f_k(r+a)\, dr, \tag{3.15}$$

Make substutition of (3.9) into (3.15), we have

$$\begin{aligned} \mathcal{F}_k(t) &= \sum_{i=k+1}^{n} \int_0^\infty e^{(\sigma+t\cos\theta)(a+r)} (a+r)^{m+i-k-1}\, dr \\ &= \sum_{i=k+1}^{n} \frac{(m+i-k-1)!}{(\sigma+t\cos\theta)^{m+i-k}}. \end{aligned}$$

On the other hand, before proceeding next step, we first introduce this assumption
H_4) there exists $\alpha > 0$, such that $|\sigma + t\cos\theta| > \alpha$.
By assumption (H_4), we obtain

$$\mathcal{F}_k(t) \leqslant \sum_{i=k+1}^{n} \frac{(m+i-k-1)!}{\alpha^{m+i-k}}. \tag{3.16}$$

Due (3.16) and (3.14), we obtain

$$\|u^m(t)\|_{\mathcal{X}} \leqslant Ne^{\omega t} \sum_{k=0}^{n-1} \|u_k\|_{\mathcal{X}},$$

for

$$t > \frac{-\sigma}{\cos\theta}, \quad m = 0, \ldots, n-1,$$

where

$$N = \frac{C}{\pi} \max_{\substack{k=0,\ldots n-1 \\ m=0,\ldots n-1}} \sum_{i=k+1}^{n} \frac{(m+i-k-1)!}{\alpha^{m+i-k}},$$

and

$$\omega = a - a\cos\theta < 2a.$$

Lemma 3.4. *If $\lambda \in \mathcal{G}(a,\theta)$, and Γ is the bound of $\mathcal{G}(a,\theta)$, we have*

$$\frac{1}{2\pi i}\int_\Gamma e^{\lambda t}\lambda^{m-k-1} d\lambda = \begin{cases} 0 & k < m, \\ \dfrac{t^{k-m}}{(k-m)!} & k \geqslant m. \end{cases}$$

Proof. Let us introduice the sets Γ_R, Δ_R, and Δ as follows:

$$\Gamma_R = \{\lambda \in \Gamma, \ \ |\lambda| \leqslant R\}.$$

$$\Delta_R = \{\lambda \in \mathbb{C} \setminus \mathcal{G}(a,\theta), \ \ |\lambda| = R\}.$$

$$\Delta = \Gamma_R \cup \Delta_R.$$

We have:

$$\int_\Delta e^{\lambda t}\lambda^{m-k-1}d\lambda = \int_{\Gamma_R} e^{\lambda t}\lambda^{m-k-1}d\lambda + \int_{\Delta_R} e^{\lambda t}\lambda^{m-k-1}d\lambda. \tag{3.17}$$

We set

$$\psi(\lambda) = e^{\lambda t}\lambda^{m-k-1}.$$

By using the parametrisation $\lambda = re^{i\omega}$, for $\lambda \in \Delta_R$, with $\omega \in [\pi - \theta, \pi + \theta]$, $0 < \theta < \dfrac{\pi}{2}$, we can write:

$$|\lambda\psi(\lambda)| = \left|e^{\lambda t}\lambda^{m-k}\right| = \left|e^{\lambda t}\right|\left|\lambda^{m-k}\right| = e^{tRe\lambda}R^{m-k} = e^{tR\cos\theta}R^{m-k}.$$

Thus, if $k < m$, it is clear that

$$\lim_{R\to\infty} e^{tR\cos\theta}R^{m-k} = 0,$$

and by virtue of Jordan lemma, we obtain

$$\lim_{R\to\infty}\int_{\Delta_R} e^{\lambda t}\lambda^{m-k-1}d\lambda = 0, \tag{3.18}$$

and

$$\int_\Delta e^{\lambda t}\lambda^{m-k-1}d\lambda = 0. \tag{3.19}$$

In light of (3.17), (3.18) and (3.19), we have

$$\begin{aligned}
\int\limits_\Gamma e^{\lambda t}\lambda^{m-k-1}d\lambda &= \lim_{R\to\infty}\int_{\Gamma_R} e^{\lambda t}\lambda^{m-k-1}d\lambda = \\
&= \lim_{R\to\infty}\int_{\Delta} e^{\lambda t}\lambda^{m-k-1}d\lambda - \lim_{R\to\infty}\int_{\Delta_R} e^{\lambda t}\lambda^{m-k-1}d\lambda = \\
&= 0 - 0 = 0.
\end{aligned}$$

On the other hand, if $k \geqslant m$, $k+1 > m$, we can write

$$\begin{aligned}
\int_\Delta e^{\lambda t}\lambda^{m-k-1}d\lambda &= \int_\Delta \frac{e^{\lambda t}}{\lambda^{k+1-m}}d\lambda = \\
&= \frac{2\pi i}{(k-m)!}\psi^{k-m}(0) = \\
&= \frac{2\pi i}{(k-m)!}t^{k-m},
\end{aligned} \tag{3.20}$$

which leads to

$$\begin{aligned}
\int_\Gamma e^{\lambda t}\lambda^{m-k-1}d\lambda &= \lim_{R\to\infty}\int_{\Gamma_R} e^{\lambda t}\lambda^{m-k-1}d\lambda = \\
&= \lim_{R\to\infty}\int_{\Delta} e^{\lambda t}\lambda^{m-k-1}d\lambda - \lim_{R\to\infty}\int_{\Delta_R} e^{\lambda t}\lambda^{m-k-1}d\lambda = \\
&= \frac{2\pi i}{(k-m)!}t^{k-m}.
\end{aligned} \tag{3.21}$$

Hence, by virtue of (3.21), we obtain

$$\frac{1}{2\pi i}\int_{\Gamma} e^{\lambda t}\lambda^{m-k-1}d\lambda = \frac{t^{k-m}}{(k-m)!}, \tag{3.22}$$

for $k \geqslant m$.

4. Main results

In this section, we use the results obtained in the previous section to obtain new appropriate conditions ensuring that of any solution u of problem (1.1), (1.2) and its derivatives $u', \ldots, u^{(p-1)}$ at any fixed point $t = t_0 > 0$ continuously dependent on all initial data (1.2).
We are now in a position to establish the following results.

Theorem 4.1. *If hypotheses (H_1) and (H_3) of Lemma 2.1. are satisfied and, further assume that*

$$\left\|\mathcal{Q}_j^{-1}(\lambda)\,A_k x\right\|_{\mathcal{X}} \leqslant C\,|\lambda|^{j-p+2}\,\|x\|_{\mathcal{X}}\,, \tag{4.1}$$

for fixed p in $\{1,\ldots,n\}$ and $k = 1,\ldots,n-1$. Then problem (1.1), (1.2) *is n-fold well-posed.*

Proof. From (1.5), we have

$$\begin{aligned}\mathcal{Q}_j^{-1}(\lambda)\,A_n &= \lambda^{j-n+1}\mathcal{Q}_j^{-1}(\lambda)\left(\mathcal{Q}_j(\lambda) - \sum_{k=j+1}^{n-1} A_k\lambda^{k-j-1}\right) = \\ &= \lambda^{j-n+1}I - \sum_{k=j+1}^{n-1}\mathcal{Q}_j^{-1}(\lambda)\,\lambda^{k-n}A_k.\end{aligned}$$

Due the condition (4.1), we get

$$\left\|\mathcal{Q}_j^{-1}(\lambda)\,A_n\right\|_{\mathcal{X}} \leqslant |\lambda|^{j-n+1} + C\sum_{k=j+1}^{n-1}|\lambda|^{k+j-n-p+2}\,. \tag{4.2}$$

From the above inequality (4.2), it can be easy to write

$$\left\|\mathcal{Q}_j^{-1}(\lambda)\,A_n\right\|_{\mathcal{X}} \leqslant |\lambda|^{j-p+1}\left(|\lambda|^{p-n} + C\sum_{k=j+1}^{n-1}|\lambda|^{k-n+1}\right). \tag{4.3}$$

Since $p \leqslant n$ and $k \leqslant n-1$, there exists $G_2 > 0$ such that:

$$\left\|\mathcal{Q}_j^{-1}(\lambda)\,A_n\right\|_{\mathcal{X}} \leqslant G_2\,|\lambda|^{j-p+1}\,. \tag{4.4}$$

This together with (4.1), leads that the hypotheses of Lemma 3.1 are verified. By virtue of Lemma 3.3, if u is a solution of the problem (1.1), (1.2), for any $\sigma > 0$

$$\|u^m(t)\|_{\mathcal{X}} \leqslant Me^{a(1+\cos\theta)t}\sum_{k=0}^{n-1}\|u_k\|_{\mathcal{X}}\,, \tag{4.5}$$

for $t > -\dfrac{\sigma}{\cos\theta}$, and $m = 0, 1\ldots, n-1$. For an arbitrary positive t, we let $t_0 = \dfrac{t}{2}$, and $\sigma_0 = -t_0\cos\theta$. We have $t > \dfrac{-\sigma_0}{\cos\theta}$, so that

$$\|u^m(t)\|_{\mathcal{X}} \leqslant Me^{at(1+\cos\theta)}\sum_{k=0}^{n-1}\|u_k\|_{\mathcal{X}}\,, \quad t > 0, \tag{4.6}$$

for $m = 0, \ldots, n-1$.
On the other hand, since $\cos\theta < 0$ which implies that $1+\cos\theta < 1$. It follows from (4.6) that:

$$\|u^m(t)\|_{\mathcal{X}} \leqslant Me^{at} \sum_{k=0}^{n-1} \|u_k\|_{\mathcal{X}}, \quad t > 0, \tag{4.7}$$

for $m = 0, \ldots, n-1$. Thus, the problem (1.1), (1.2) is n-fold well-posed.

Theorem 4.2. *If the conditions of Theorem 3.1 are satisfied, then the problem* (1.1), (1.2) *is p-fold exponentially well-posed.*

Proof. The remaining part of the proof is similar to Theorem 3.1. There exists $M' > 0$, such that

$$\|u^m(t)\|_{\mathcal{X}} \leqslant M'e^{at} \sum_{k=0}^{n-1} \|u_k\|_{\mathcal{X}}, \tag{4.8}$$

for $t \geqslant 1$, and $m = 0, 1 \ldots, n-1$. Suppose now $t \in \,]0,1[$, from (1.3), (1.4) and (1.5), we can get

$$\mathcal{Q}_k(\lambda) = \frac{1}{\lambda^{k+1}} \left(\mathcal{P}(\lambda) - \mathcal{P}_k(\lambda)\right), \tag{4.9}$$

so that

$$\begin{aligned} \sum_{k=0}^{n-2} \mathcal{P}^{-1}(\lambda)\, \mathcal{Q}_k(\lambda)\, u_k &= \sum_{k=0}^{n-2} \lambda^{-k-1} \mathcal{P}^{-1}(\lambda) \left(\mathcal{P}(\lambda) - \mathcal{P}_k(\lambda)\right) u_k = \\ &= \sum_{k=0}^{n-2} \lambda^{-k-1} u_k - \sum_{k=0}^{n-2} \lambda^{-k-1} \mathcal{P}^{-1}(\lambda)\, \mathcal{P}_k(\lambda)\, u_k. \end{aligned} \tag{4.10}$$

Hence, we can get

$$\begin{aligned} u^m(t) &= -\frac{1}{2\pi i} \int_\Gamma \lambda^m e^{\lambda t} \sum_{k=0}^{n-2} \mathcal{P}^{-1}(\lambda)\, \mathcal{Q}_k(\lambda)\, u_k d\lambda = \\ &= -\frac{1}{2\pi i} \int_\Gamma \lambda^m e^{\lambda t} \sum_{k=0}^{n-2} \mathcal{P}^{-1}(\lambda)\, \mathcal{Q}_k(\lambda)\, u_k d\lambda - \frac{1}{2\pi i} \int_\Gamma \lambda^m e^{\lambda t} \mathcal{P}^{-1}(\lambda)\, A_n u_{n-1} d\lambda = \\ &= -\frac{1}{2\pi i} \int_\Gamma \sum_{k=0}^{n-2} \lambda^{m-k-1} e^{\lambda t} u_k d\lambda + \frac{1}{2\pi i} \int_\Gamma \sum_{k=0}^{n-2} \lambda^{m-k-1} e^{\lambda t} \mathcal{P}^{-1}(\lambda)\, \mathcal{P}_k(\lambda)\, u_k d\lambda - \\ &- \frac{1}{2\pi i} \int_\Gamma \lambda^m e^{\lambda t} \mathcal{P}^{-1}(\lambda)\, A_n u_{n-1} d\lambda. \end{aligned} \tag{4.11}$$

From (4.11) and (3.5), we get

$$\begin{aligned} u^m(t) &= -\frac{1}{2\pi i} \int_\Gamma e^{\lambda t} \sum_{k=0}^{n-2} \lambda^{m-k-1} u_k d\lambda + \\ &+ \frac{1}{2\pi i} \int_\Gamma e^{\lambda t} \sum_{k=0}^{n-2} \lambda^{m-k-j-2} \mathcal{R}_j(\lambda)\, \mathcal{Q}_j^{-1}(\lambda)\, \mathcal{P}_k(\lambda)\, u_k d\lambda - \\ &- \frac{1}{2\pi i} \int_\Gamma \lambda^{m-j-1} e^{\lambda t} \mathcal{R}_j(\lambda)\, \mathcal{Q}_j^{-1}(\lambda)\, A_n u_{n-1} d\lambda. \end{aligned} \tag{4.12}$$

In view of Lemma 3.4, we obtain

$$\begin{aligned} u^{m}(t) &= -\sum_{k=m}^{n-2} \frac{t^{k-m}}{(k-m)!} u_k + \\ &+ \frac{1}{2\pi i} \int_{\Gamma} e^{\lambda t} \sum_{k=0}^{n-2} \lambda^{m-k-j-2} \mathcal{R}_j(\lambda) \mathcal{Q}_j^{-1}(\lambda) \mathcal{P}_k(\lambda) u_k d\lambda - \\ &- \frac{1}{2\pi i} \int_{\Gamma} \lambda^{m-j-1} e^{\lambda t} \mathcal{R}_j(\lambda) \mathcal{Q}_j^{-1}(\lambda) A_n u_{n-1} d\lambda. \end{aligned} \tag{4.13}$$

Let Γ_t be the sector of the complex plane defined by

$$\Gamma_t = \left\{ \lambda = \frac{a}{t} + re^{i\varphi},\ |\arg| \leqslant \theta \right\},$$

for $t \in]0,1[$, where a and θ are defined in $\mathcal{G}(a,\theta)$.
The functions under the sign of integration in (4.13) are analytic, so we can write

$$\begin{aligned} u^{m}(t) &= -\sum_{k=m}^{n-2} \frac{t^{k-m}}{(k-m)!} u_k + \\ &+ \frac{1}{2\pi i} \int_{\Gamma_t} e^{\lambda t} \sum_{k=0}^{n-2} \lambda^{m-k-j-2} \mathcal{R}_j(\lambda) \mathcal{Q}_j^{-1}(\lambda) \mathcal{P}_k(\lambda) u_k d\lambda - \\ &- \frac{1}{2\pi i} \int_{\Gamma_t} \lambda^{m-j-1} e^{\lambda t} \mathcal{R}_j(\lambda) \mathcal{Q}_j^{-1}(\lambda) A_n u_{n-1} d\lambda. \end{aligned} \tag{4.14}$$

If we set $\mathcal{Z} = \lambda t$. Due (4.14), it is easy to verify that

$$\begin{aligned} u^{m}(t) &= -\sum_{k=m}^{n-2} \frac{t^{k-m}}{(k-m)!} u_k + \\ &+ \frac{1}{2\pi i} \int_{\Gamma} \frac{e^{\lambda t}}{t} \sum_{k=0}^{n-2} \left(\frac{\mathcal{Z}}{t}\right)^{m-k-j-2} \mathcal{R}_j\left(\frac{\mathcal{Z}}{t}\right) \mathcal{Q}_j^{-1}\left(\frac{\mathcal{Z}}{t}\right) \mathcal{P}_k\left(\frac{\mathcal{Z}}{t}\right) u_k d\mathcal{Z} - \\ &- \frac{1}{2\pi i} \int_{\Gamma} \frac{e^{\lambda t}}{t} \left(\frac{\mathcal{Z}}{t}\right)^{m-j-1} \mathcal{R}_j\left(\frac{\mathcal{Z}}{t}\right) \mathcal{Q}_j^{-1}\left(\frac{\mathcal{Z}}{t}\right) A_n u_{n-1} d\mathcal{Z}. \end{aligned}$$

This leads to the following estimate

$$\begin{aligned} \|u^{m}(t)\|_{\mathcal{X}} &\leqslant \sum_{k=m}^{n-2} \frac{t^{k-m}}{(k-m)!} \|u_k\|_{\mathcal{X}} + \\ &+ \frac{G_0}{2\pi} \int_{\Gamma} \frac{e^{Re\mathcal{Z}}}{t} \sum_{k=0}^{n-2} \left|\frac{\mathcal{Z}}{t}\right|^{m-k-j-2} \left\| \mathcal{Q}_j^{-1}\left(\frac{\mathcal{Z}}{t}\right) \mathcal{P}_k\left(\frac{\mathcal{Z}}{t}\right) \right\|_{\mathcal{X}} \|u_k\|_{\mathcal{X}} d\mathcal{Z} + \\ &+ \frac{G_0}{2\pi} \int_{\Gamma} \frac{e^{Re\mathcal{Z}}}{t} \left|\frac{\mathcal{Z}}{t}\right|^{m-j-1} \left\| \mathcal{Q}_j^{-1}\left(\frac{\mathcal{Z}}{t}\right) A_n \right\|_{\mathcal{X}} \|u_{n-1}\|_{\mathcal{X}} d\mathcal{Z}. \end{aligned} \tag{4.15}$$

Now, we estimate the second and third terms on the right side of (4.15),
Firstly, by using relation (4.1), we obtain

$$\begin{aligned} \left\| \mathcal{Q}_j^{-1}\left(\frac{\mathcal{Z}}{t}\right)\mathcal{P}_k\left(\frac{\mathcal{Z}}{t}\right)\right\|_{\mathcal{X}} = \left\| \mathcal{Q}_j^{-1}\left(\frac{\mathcal{Z}}{t}\right)\sum_{s=0}^{k} A_s\left(\frac{\mathcal{Z}}{t}\right)^s\right\|_{\mathcal{X}} &\leqslant \sum_{s=0}^{k}\left|\frac{\mathcal{Z}}{t}\right|^s \left\|\mathcal{Q}_j^{-1}A_s\right\|_{\mathcal{X}} \leqslant \\ &\leqslant C\sum_{s=0}^{k}\left|\frac{\mathcal{Z}}{t}\right|^{j-p+2+s}. \end{aligned}$$

Thus

$$\begin{aligned} \sum_{k=0}^{n-2}\left|\frac{\mathcal{Z}}{t}\right|^{m-k-j-2}\left\| \mathcal{Q}_j^{-1}\left(\frac{\mathcal{Z}}{t}\right)\mathcal{P}_k\left(\frac{\mathcal{Z}}{t}\right)\right\|_{\mathcal{X}} &\leqslant C\sum_{k=0}^{n-2}\sum_{s=0}^{k}\left|\frac{\mathcal{Z}}{t}\right|^{m-k+s-p} = \\ &= C\sum_{k=0}^{n-2}\sum_{s=0}^{k}\left|\frac{t}{\mathcal{Z}}\right|^{p-m+k-s}. \end{aligned} \tag{4.16}$$

Secondly, by the same way, we have

$$\left\| \mathcal{Q}_j^{-1}\left(\frac{\mathcal{Z}}{t}\right)A_n\right\|_{\mathcal{X}} \leqslant G_2\left|\frac{\mathcal{Z}}{t}\right|^{j+1-p}.$$

Therefore

$$\left|\frac{\mathcal{Z}}{t}\right|^{m-j-1}\left\| \mathcal{Q}_j^{-1}\left(\frac{\mathcal{Z}}{t}\right)A_n\right\|_{\mathcal{X}} \leqslant G_2\left|\frac{\mathcal{Z}}{t}\right|^{m-p} = G_2\left|\frac{t}{\mathcal{Z}}\right|^{p-m}. \tag{4.17}$$

Substituting (4.16) and (4.17) into (4.15), we obtain

$$\begin{aligned} \left\|u^m(t)\right\|_{\mathcal{X}} &\leqslant \sum_{k=m}^{n-2}\frac{t^{k-m}}{(k-m)!}\left\|u_k\right\|_{\mathcal{X}} + \\ &+ \frac{G_3}{2\pi}\int_\Gamma \frac{e^{Re\mathcal{Z}}}{|\mathcal{Z}|}\sum_{k=0}^{n-2}\sum_{s=0}^{k}\left(\frac{t}{|\mathcal{Z}|}\right)^{p-m+k-s-1}\left\|u_k\right\|_{\mathcal{X}} d\mathcal{Z} + \\ &+ \frac{G_4}{2\pi}\int_\Gamma \frac{e^{Re\mathcal{Z}}}{t}\frac{t^{p-m}}{|\mathcal{Z}|^{p-m}}\left\|u_{n-1}\right\|_{\mathcal{X}} d\mathcal{Z}, \end{aligned}$$

where $G_3 = CG_0$, and $G_4 = G_2G_0$. This leads

$$\begin{aligned} \left\|u^m(t)\right\|_{\mathcal{X}} &\leqslant \sum_{k=m}^{n-2}\frac{t^{k-m}}{(k-m)!}\left\|u_k\right\|_{\mathcal{X}} + \\ &+ \frac{G_3}{2\pi}\int_\Gamma \frac{e^{Re\mathcal{Z}}}{t}\sum_{k=0}^{n-2}\sum_{s=0}^{k}\left(\frac{t}{|\mathcal{Z}|}\right)^{p-m+k-s}\left\|u_k\right\|_{\mathcal{X}} d\mathcal{Z} + \\ &+ \frac{G_4}{2\pi}\int_\Gamma \frac{e^{Re\mathcal{Z}}}{t}\frac{t^{p-m}}{|\mathcal{Z}|^{p-m}}\left\|u_{n-1}\right\|_{\mathcal{X}} d\mathcal{Z}. \end{aligned}$$

By means of parametrisation $\mathcal{Z} = at + tre^{i\theta}$, we obtain

$$\begin{aligned}\|u^m(t)\|_{\mathcal{X}} &\leqslant \sum_{k=m}^{n-2} \frac{t^{k-m}}{(k-m)!} \|u_k\|_{\mathcal{X}} + \\ &+ \frac{G_3}{2\pi} \int_0^\infty e^{at+tr\cos\theta} \sum_{k=0}^{n-2} \sum_{s=0}^{k} \left(\frac{1}{a+r}\right)^{p-m+k-s} \|u_k\|_{\mathcal{X}}\, dr + \\ &+ \frac{G_4}{2\pi} \int_0^\infty e^{at+tr\cos\theta} \left(\frac{1}{a+r}\right)^{p-m} \|u_{n-1}\|_{\mathcal{X}}\, dr.\end{aligned}$$

Therefore, we conclude that

$$\begin{aligned}\|u^m(t)\|_{\mathcal{X}} &\leqslant \sum_{k=m}^{n-2} \frac{t^{k-m}}{(k-m)!} \|u_k\|_{\mathcal{X}} + \\ &+ \frac{G_3}{2\pi} \int_0^\infty e^{at+tr\cos\theta} \sum_{k=0}^{n-2} \sum_{s=0}^{k} (a+r)^{m-p+s-k} \|u_k\|_{\mathcal{X}}\, dr + \\ &+ \frac{G_4}{2\pi} \int_0^\infty e^{at+tr\cos\theta} (a+r)^{m-p} \|u_{n-1}\|_{\mathcal{X}}\, dr.\end{aligned}$$

A simple computation shows that

$$\begin{aligned}\|u^m(t)\|_{\mathcal{X}} &\leqslant \sum_{k=m}^{n-2} \frac{t^{k-m}}{(k-m)!} \|u_k\|_{\mathcal{X}} + \\ &+ \frac{G_3}{2\pi} e^{(a-a\cos\theta)t} \int_0^\infty e^{t\cos\theta(a+r)} \sum_{k=0}^{n-2} \sum_{s=0}^{k} (a+r)^{m-p+s-k} \|u_k\|_{\mathcal{X}}\, dr + \\ &+ \frac{G_4}{2\pi} e^{(a-a\cos\theta)t} \int_0^\infty e^{t\cos\theta(a+r)} (a+r)^{m-p} \|u_{n-1}\|_{\mathcal{X}}\, dr. \qquad (4.18)\end{aligned}$$

Next, we estime the second and the third terms in the right side of (4.18).
For $m < p$, we have $m - p + s - k < 0$, so

$$(a+r)^{m-p+s-k} < (a)^{m-p+s-k}.$$

If we put $M_k = \max\limits_{s=0,\dots k} a^{m-p+s-k}$, we obtain

$$\int_0^\infty e^{t\cos\theta(a+r)} \sum_{s=0}^{k} (a+r)^{m-p+s-k}\, dr \leqslant (k+1) M_k \int_0^\infty e^{t\cos\theta(a+r)} dr = \frac{-(k+1) M_k e^{ta\cos\theta}}{t\cos\theta}.$$

In a similar way, we can write

$$\int_0^\infty e^{t\cos\theta(a+r)} (a+r)^{m-p}\, dr \leqslant a^{m-p} \int_0^\infty e^{(a+r)t\cos\theta} dr = \frac{-a^{m-p} e^{ta\cos\theta}}{t\cos\theta}.$$

Finally

$$\begin{aligned}\|u^m(t)\|_{\mathcal{X}} \leqslant \sum_{k=m}^{n-2} \frac{t^{k-m}}{(k-m)!} \|u_k\|_{\mathcal{X}} &+ \frac{G_3}{2\pi} \sum_{k=0}^{n-2} \frac{-(k+1) M_k e^{ta\cos\theta}}{t\cos\theta} \|u_k\|_{\mathcal{X}} + \\ &+ \frac{G_4}{2\pi} \left(\frac{-a^{m-p}}{t\cos\theta}\right) e^{at} \|u_{n-1}\|_{\mathcal{X}}.\end{aligned}$$

Since $t > 0$ and $\cos\theta < 0$, we choose that

$$G_5 = \frac{G_3}{2\pi} \max_{k=0,\dots n-2} \frac{-(k+1) M_k e^{\cos\theta}}{t\cos\theta},$$

and

$$G_6 = \frac{G_4}{2\pi}\left(\frac{-a^{m-p}}{t\cos\theta}\right),$$

so

$$\|u^m(t)\|_{\mathcal{X}} \leqslant \sum_{k=m}^{n-2} \frac{t^{k-m}}{(k-m)!}\|u_k\|_{\mathcal{X}} + G_5 e^{ta}\sum_{k=0}^{n-2}\|u_k\|_{\mathcal{X}} + G_6 e^{at}\|u_{n-1}\|_{\mathcal{X}}.$$

On the other hand, we have

$$\sum_{k=m}^{n-2} \frac{t^{k-m}}{(k-m)!}\|u_k\|_{\mathcal{X}} \leqslant \sum_{k=m}^{n-2}\|u_k\|_{\mathcal{X}}.$$

If we put $R = \max(1, G_5, G_6)$, we obtain

$$\|u^m(t)\|_{\mathcal{X}} \leqslant Re^{ta}\sum_{k=0}^{n-1}\|u_k\|_{\mathcal{X}}.$$

This together with (4.8), it follows that the problem (1.1), (1.2) is p-fold exponentially well-posed. □

Remark 4.3. *The results presented in this paper improve and extend the main result proved in [18] under appropriate conditions. As far as we know, sufficient conditions for the p-fold well-posedness of higher-order abstract Cauchy problem expressed in terms of decay of some auxiliary pencils shown in (1.4) and (1.5) considered in the present paper have not been investigated yet. For this reason, in this paper we make the first attempt to fill this gap. The method employed in this paper is different from those in related literature (Vlasenko et al [18]).*

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О р-кратной корректности абстрактной задачи Коши высшего порядка

Хабиба Тоуми
Лаборатория LAMAHIS, Университет Баджи Мохтар
Аннаба 23000, Алжир
Ахмед Нуар
Лаборатория LAMAHIS, Университет 20 Aŷт 1955
Скикда 21000, Алжир

Аннотация. В данной статье мы устанавливаем достаточные условия p-кратной корректности абстрактной задачи Коши высокого порядка. Эти условия выражаются через затухание некоторых вспомогательных пучков, полученных из характеристического пучка рассматриваемого операционного дифференциального уравнения. В частности, эта статья совершенствует важную и интересную работу.

Ключевые слова: абстрактные задачи Коши, интегрированные полугруппы, корректность.